(e,h_1,h_2)	Reversor	$q=(x,y,z),\ p=(u,v,w)$
$(0,\lambda_1,\lambda_2)$	R	$x_l^2=\dfrac{a_l\,(a_l-\lambda_1)\,(a_l-\lambda_2)}{(a_l-a_m)\,(a_l-a_n)}$, $p=\sqrt{\dfrac{a\,b\,c}{\lambda_1\,\lambda_2}}\left(\dfrac{x}{a},\dfrac{y}{b},\dfrac{z}{c}\right)$
$e=0,\ \{h_1,h_2\}=\{a_l,\lambda_j\}$	R_{x_l}	$x_l=0,\ x_m^2=\dfrac{a_m\,(a_m-\lambda_j)}{a_m-a_n}$, $u_l^2=\dfrac{a_l-\lambda_k}{a_l},\ u_m=a_n x_m\sqrt{\dfrac{\lambda_k}{abc\lambda_j}}$
$e=0,\ \{h_1,h_2\}=\{a_m,a_n\}$	$R_{x_m x_n}$	$x_l=\pm\sqrt{a_l},\ x_m=x_n=0$, $u_l^2=\dfrac{\lambda_1\,\lambda_2}{a_m\,a_n},\ u_m^2=\dfrac{(a_m-\lambda_1)\,(a_m-\lambda_2)}{a_m\,(a_m-a_n)}$
$\{e,h_1,h_2\}=\{a_l,\lambda_1,\lambda_2\}$	$f\circ R_{x_l}$	$x_l^2=\dfrac{a_l\,\lambda_1\,\lambda_2}{a_m a_n},\ x_m^2=\dfrac{(a_m-\lambda_1)\,(a_m-\lambda_2)}{a_m-a_n}$, $u_l=\pm1,\ u_m=0$
$\{e,h_1,h_2\}=\{a_m,a_n,\lambda_j\}$	$f\circ R_{x_m x_n}$	$x_l^2=a_l-\lambda_k,\ x_m=u_m\sqrt{\dfrac{a_m\,a_n\,\lambda_k}{a_l\,\lambda_j}}$, $u_l=0,\ u_m^2=\dfrac{a_m-\lambda_j}{a_m-a_n}$
(c,b,a)	$f\circ R_{xyz}$	$u_l^2=\dfrac{(a_l-\lambda_1)\,(a_l-\lambda_2)}{(a_l-a_m)\,(a_l-a_n)}$, $q=\sqrt{\dfrac{a\,b\,c}{\lambda_1\,\lambda_2}}\,(u,v,w)$

Table 1. Likewise, we present the corresponding (q,p) formulae.

Conclusiones

Trozos

Symmetries through elliptic coordinates

We give a complete classification of the symmetry sets, in connection with the vertexes in elliptic coordinates they come from. The next proposition is summarized in table ?.

Lemma 1. (Characterization of reversible maps) *A map f is reversible if and only if it can be factorized as the composition of two involutions, in which case both of them are reversors of f.*

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Proof. Let us assume that f is $\tilde{r}$-reversible. Then $\hat{r}=f\circ\tilde{r}=\tilde{r}\circ f^{-1}$ is another reversor, because:

1. $f\circ\hat{r}=f\circ\tilde{r}\circ f^{-1}=\hat{r}\circ f^{-1}$,
2. $\hat{r}^2=\hat{r}\circ\hat{r}=\tilde{r}\circ f^{-1}\circ f\circ\tilde{r}=\tilde{r}^2=\langle\text{Identity}\rangle$.

Therefore, the map $f=f\circ\tilde{r}^2=\hat{r}\circ\tilde{r}$ is the composition of two involutions.
On the other hand, if $f=\hat{r}\circ\tilde{r}$ and $\hat{r}^2=\tilde{r}^2=\langle\text{Identity}\rangle$, then:

1. $f\circ\tilde{r}\circ f=\hat{r}\circ\tilde{r}^2\circ\hat{r}\circ\tilde{r}=\hat{r}^2\circ\tilde{r}=\tilde{r}$,
2. $f\circ\hat{r}\circ f=\hat{r}\circ\tilde{r}\circ\hat{r}^2\circ\tilde{r}=\hat{r}\circ\tilde{r}^2=\hat{r}$,

so both involutions $\hat{r}$ and $\tilde{r}$ are reversors of the map f. □ | El lema, Â¿estÃą publicado en alguna parte?|SegÃžn [?], aparece en Birkhoff, G.D., (1915). The restricted problem of three bodies. Rend. Circ. Mat. Palermo 39, 265-334.⟩

Proposition 2. *Let us denote $\mathcal{C}_T$ as the cuboid:*

$$\mathcal{C}_T=\begin{cases}[0,\lambda_1]\times[c,\lambda_2]\times[b,a], & T=\text{EH1},\\ [0,c]\times[\lambda_1,\lambda_2]\times[b,a], & T=H1H1,\\ [0,\lambda_1]\times[c,b]\times[\lambda_2,a], & T=\text{EH2},\\ [0,c]\times[\lambda_1,b]\times[\lambda_2,a], & T=H1H2,\end{cases}$$

where elliptic coordinates take place in a billiard trajectory. Then, every vertex of $\mathcal{C}_T$ corresponds to a SO.