

Roots of a Polynomial

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In this example I'll try to find the real roots of the polynomial $x^3 - 12.5x^2 + 1.353$ first using Mathcad and then using Texmacs with the computer algebra system Maxima.

I made this document using Texmacs, with the Maxima and Octave plugins, to show some of the things you can do with Texmacs and to compare it with Mathcad.

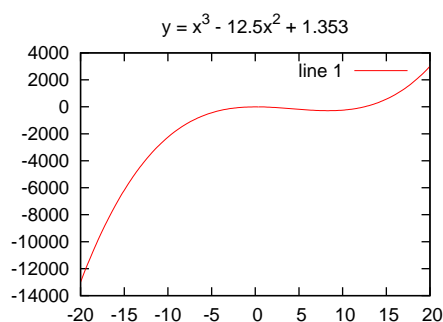
Mathcad

*This is **not** Mathcad running in Texmacs, it simply shows how you might solve the problem in Mathcad. The equations are simply inactive text and the graphs were plotted in Texmacs.*

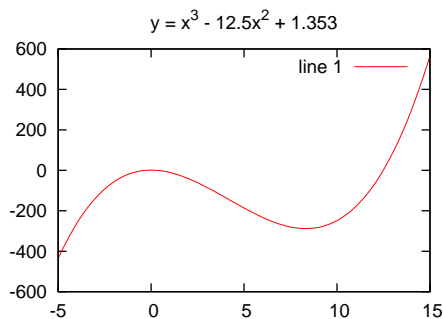
The first bit of maths I type into my Mathcad worksheet is the equation to be solved.

$$f(x) := x^3 - 12.5x^2 + 1.353$$

I'll use Mathcad's "solver". You have to supply an initial value for x (your "guess") and the solver tries to find a nearby root. If you don't know what would be a good guess then it helps to plot a graph of the function $f(x)$. I'll start by plotting over the range $-20 \leq x \leq 20$.



It's hard to see where the roots are so I'll plot a smaller range, say $-5 \leq x \leq 15$.



It looks like there is a root near $x = 12$ and possibly others near $x = 0$. I'll use -2, 2 and 12 as my guesses for the roots.

Lets try the solver with my first guess (I've shown Mathcad's output in red).

$$x := -2$$

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$h := \text{root}(f(x), x) \quad h = -0.325$ (first root)

Looks good. Now lets try to find the second and third roots.

$x := 2$ (second guess)

$h := \text{root}(f(x), x) \quad h = 0.333$ (second root)

$x := 12$ (third guess)

$h := \text{root}(f(x), x) \quad h = 12.491$ (third root)

I did a certain amount of graph plotting to choose my guesses but I got there in the end!

Texmacs + Maxima

Now lets use Texmacs with Maxima to solve the problem. I'll use Maxima's *realroots* function to find the roots, together with *float* which converts the answer to a simple numeric form of the kind Mathcad would produce.

Select *Insert -> Session -> Maxima* to start a new Maxima session. When you start a Maxima session it generates some introductory text which I have deleted. Maxima puts a label at the start of each input and output line, shown in red, so that you can refer to an earlier result by its label. My maths typing appears in blue and Maxima's responses appear in black. Here is my Maxima session:

First I type the equation to be solved

(%i1) $f(x) := x^3 - 12.5x^2 + 1.353$

(%o1) $f(x) := x^3 - 12.5x^2 + 1.353$

Then I ask Maxima to find the real roots and print them as simple floating point numbers

(%i2) $\text{float}(\text{realroots}(f(x)))$

(%o2) $[x = -0.32480558753014, x = 0.33347681164742, x = 12.4913287460804]$

Well, I have to admit that was actually easier than Mathcad!

Further Notes

Solving Equations with Maxima

In the above example I used *realroots* to find the roots of an equation but Maxima also has other tools for this. There is *allroots* which can find complex roots too and there is *solve* which will try to solve the equation algebraically and give an exact answer. *Solve* is very good but it can produce quite large expressions and if you just want simple numerical answers then *allroots* and *realroots* are what you need, together with *float* to produce the answer in the form of simple floating point numbers.

Mathematical Notation

You probably noticed that there don't seem to be any multiplication symbols in the above equations, for example, between "12.5" and "x" in the expression "12.5 x" (Mathcad, by comparison, would show a dot between the two). Well, actually, there is a multiplication symbol, which you enter by typing an asterisk, but Texmacs only shows it as a small additional space. Nonetheless, it is necessary because it tells Texmacs that these are two separate quantities that are to be multiplied together rather than a single variable called "12.5x". Hence Texmacs knows that we mean multiplication but follows the normal convention of not showing it explicitly.

To a large degree you can use mathematical notation to type your input within a session but there will be some constraints and irregularities due to the command syntax of the particular maths plugin you are using (Maxima, Octave, R or whatever), hence the need for things like $:=$ when writing an equation in a Maxima session. My impression so far is that it is no worse than the differences between Mathcad and conventional mathematical notation.

Using Octave to plot some graphs

Below is an Octave session that I used to make the graph plots which appear earlier in the document. I put this working here to avoid cluttering the main part of the document then copied and pasted the graphics to where I wanted them. Octave is a very sophisticated numerical package, but it is also handy for creating simple, nice looking, graph plots within your document. I have written comments before each step involved in making the first graph so that you can see how it works.

Select *Insert -> Session -> Octave* to start a new Octave session. When you start an Octave session it generates some introductory text which I have deleted. Plots are sometimes followed by a spurious warning message which I have also deleted. As you can see, in spite of these rough edges, Octave is quite usable in Texmacs. Here is my Octave session:

Set a title for the graph (I found it better not to use mathematical input mode for this)

```
octave> title('y = x^3 - 12.5x^2 + 1.353')
```

Create a vector x who's elements are a sequence of values between -20 and 20 at intervals of 0.2.

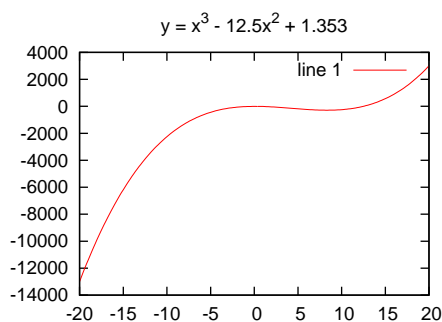
```
octave> x = [-20:0.2:20]
```

Calculate a vector of y values from the vector x. Note the “.” after each x. This tells Octave to perform a scalar operation using each element of the vector x rather than a vector operation using the whole of x (for example the term $x.^3$ gives a vector, each element of which is the cube of the corresponding element of x; hence when we write $x.^3 - 12.5 x.^2$ we are in fact subtracting two vectors). This line is equivalent to $y_i := (x_i)^3 - 12.5 (x_i)^2 + 1.353$ in Mathcad.

```
octave> y = x.^3 - 12.5 x.^2 + 1.353
```

Now we can plot the graph

```
octave> plot(x, y)
```



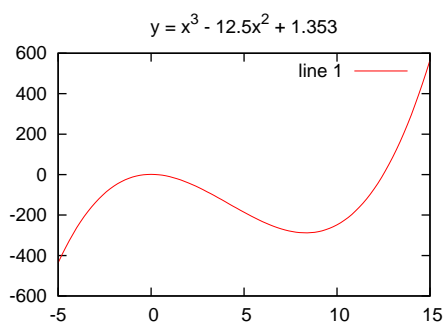
And now for the remaining graphs...

```
octave> x = [-5:0.2:15]
```

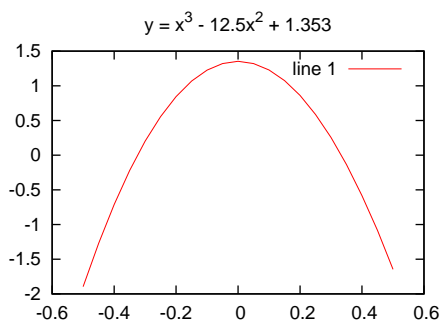
```
octave> y = x.^3 - 12.5 x.^2 + 1.353
```

```
octave> plot(x, y)
```

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```
octave> x = [-0.5:0.05:0.5]
octave> y = x.^3 - 12.5*x.^2 + 1.353
octave> plot(x, y)
```



```
octave>
```

Software Used

Debian GNU/Linux (version 4.0r4 x86_64) with the following packages:

Texmacs 1.0.7 compiled from source (although the pre-compiled package in the Debian repository, which is version 1.0.6-10, also works well)

Maxima (version 5.10.0-6)

Octave (version 2.1.73-13 - a Texmacs plugin for newer versions is under development)

R (version 2.4.0.20061125-1)

GNUplot (version 4.0.0) - used by Maxima and Octave for graph plotting.