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by Christoph Schiller and Tae-Wong SEO with special thanks to your mom (Jeong-Gi JEONG), our kids (Kang-Yeon and Yu-Jin) and everyone else who has supported this article!

For Christoph Schiller's wife please check the acknowledgements page.

Abstract

This is a test file to have plagiarized a bunch of content from the Motion Mountain books.

1 Interference: how can a wave be made up of particles?

> Die ganzen fünfzig Jahre bewusster Grübelei haben mich der Antwort auf die Frage 'Was sind Lichtquanten?' nicht näher gebracht. Heute glaubt zwar jeder Lump er wisse es, aber er täuscht sich.
>
> Albert Einstein, 1951

If a light wave is made of particles, one must be able to explain each and every wave property in terms of photons. The experiments mentioned above already hint that this is possible only because photons are quantum particles. Let us take a more detailed look at this argument.

Light can cross other light undisturbed. This observation is not hard to explain with photons; since photons do not interact with each other, and are point-like, they 'never' hit each other. In fact, there is an extremely small positive probability for their interaction, as will be found below, but this effect is not observable in everyday life.

But a problem remains. If two light beams of identical frequency and fixed phase relation cross, we observe alternating bright and dark regions: so-called *interference fringes*.*

How do these interference fringes appear? How can it be that photons are not detected in the dark regions? We already know the only possible answer: the brightness at a given place corresponds to the probability that a photon will arrive there. The fringes imply:

(Parts of the text are obfuscated.)

> A mathematician is a machine that transforms coffee into theorems.
>
> Paul Erdős (b. 1913 Budapest, d. 1996 Warsaw)

Mathematical concepts can all be expressed in terms of 'sets' and 'relations.' any fundamental concepts were presented in the last chapter. Why does mathematics, given this simple basis, grow into a passion for certain people?

The following pages present a few more advanced concepts as simply and vividly as possible, for all those who want to smell the passion for mathematics.

In particular, in this appendix we shall introduce the simplest algebraic structures. The appendix in the next volume will present some more involved algebraic structures and the most important topological structures; the third basic type of mathematical structures, order structures, are not so important in physics.

Mathematicians are concerned not only with the exploration of concepts, but also with their classification. Whenever a new mathematical concept is introduced, mathematicians try to classify all the possible cases and types. This has been achieved most spectacularly for the different types of numbers, for finite simple groups and for many types of spaces and manifolds.

1 Magnetic fields

All experiments show that the magnetic field has a given direction in space, and a magnitude common to all (resting) observers, whatever their orientation. We are thus tempted to describe the magnetic field by a vector. However, this would be wrong, since a magnetic field does not behave like an arrow when placed before a mirror. Imagine that a system produces a magnetic field directed to the right. You can take any system, a coil, a machine, etc. Now build or imagine a second system that is the exact mirror version of the first: a mirror coil, a mirror machine, etc. The magnetic system produced by the mirror system does not point to the left, as maybe you expected: it still points to the right.

(Check by yourself.) In simple words, magnetic fields do *not* fully behave like arrows. In other words, it is not completely correct to describe a magnetic field by a vector $\boldsymbol{B} = (B_x, B_y, B_z)$, as vectors behave like arrows. The magnetic field is a *pseudovector*; angular momentum and torque are also examples of such quantities. The precise way is to describe the magnetic field by the quantity[1]

$$\boldsymbol{B} = \begin{pmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{pmatrix}, \tag{1}$$

called an *antisymmetric tensor*.

The *magnetic field* is defined by the acceleration it imparts on moving charges. This acceleration is observed to follow

$$\boldsymbol{a} = \frac{e}{m}\boldsymbol{v} \times \boldsymbol{B} \tag{2}$$

a relation which is often called Lorentz acceleration, after the important Dutch physicist Hendrik A. Lorentz[2] who first stated it clearly.[3]

The Lorentz acceleration is the effect at the root of any electric motor. An electric motor is a device that uses a magnetic field as efficiently as possible to accelerate charges flowing in a wire. Through the motion of the charges, the wire is then also moved. In an electric motor, electricity is thus transformed into magnetism and then into motion. The first efficient electric motors were built already in the 1830s.

Like for the electric field, we need to know how the *strength* of a magnetic field is determined by a moving charge. Experiments such as Oersted's show that the magnetic field is due to moving charges, and that a point-like charge moving with velocity $\boldsymbol{v}$ produces a field $\boldsymbol{B}$ given by

TABLE 12 Some sensors for static and quasistatic magnetic fields.

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1. The quantity $\boldsymbol{B}$ was not called the 'magnetic field' until recently. We follow here the modern, logical definition, which supersedes the traditional one, where $\boldsymbol{B}$ was called the 'magnetic flux density' or 'magnetic induction' and another quantity, $\boldsymbol{H}$, was called– incorrectly, but for over a century – the magnetic field. This quantity $\boldsymbol{H}$ will not appear in this walk, but it is important for the description of magnetism in materials.

2. Hendrik A. Lorentz, (b. 1853 Arnhem, d. 1928 Haarlem). For more details on his biography, see the volume on relativity.

3. The expression $\boldsymbol{v} \times \boldsymbol{B}$ is the vector product of the two vectors. The most practical way to calculate the vector product $\boldsymbol{v} \times \boldsymbol{B}$ component by component is given by the determinant

$$\boldsymbol{v} \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{e}_x & v_x & B_x \\ \boldsymbol{e}_y & v_y & B_y \\ \boldsymbol{e}_z & v_z & B_z \end{vmatrix} \text{or, more sloppily } \boldsymbol{v} \times \boldsymbol{B} = \begin{vmatrix} + & - & + \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}. \tag{3}$$

This is easy to remember and easy to perform, both with letters and with numerical values. (Here, $\boldsymbol{e}_x$ is the unit basis vector in the x direction.) Written out, it is equivalent to the relation

$$\boldsymbol{v} \times \boldsymbol{B} = (v_y B_z - B_y v_z, B_x v_z - v_x B_z, v_x B_y - B_x v_y) \tag{4}$$

which is harder to remember.

The Lorentz relation is also called the *Laplace acceleration*. It defines the magnitude and the direction of the magnetic field $\boldsymbol{B}$. The unit of the magnetic field is called tesla and is abbreviated T. One has $1\text{T} = 1\ \text{N s/C m} = 1\ \text{V s/m}^2 = 1\ \text{V s}^2/\text{A m}$.

The definition of the magnetic field again assumes, like that of the electric field, that the test charge q is so small that it does not disturb the field $\boldsymbol{B}$ to be measured. Again, we ignore this issue, which means that we ignore all quantum effects, until later in our adventure.

The definition of the magnetic field also assumes that space-time is flat, and it ignores all issues due to space-time curvature.

Does the definition of magnetic field given here assume a charge speed much lower than that of light?

Measurement	Sensor	Range
Voltage	Hall probe	up to many T
Induced electromotive force (voltage)	doves	from a few T
Bone growth stimulation	piezoelectricity and magnetostriction of bones	from 50 mT
Induced electromotive force (voltage)	human nerves	from a few T
Sensations in thorax and shoulders	human nerves	strong switched gradients
Sharks	induced voltage when waving left to right	a few nT
Plants	unclear	small effects on growth

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} q \frac{\boldsymbol{v} \times \boldsymbol{r}}{r^3} \text{where} \frac{\mu_0}{4\pi} = 10^{-7} \mathrm{N/A^2}. \tag{5}$$

This is called *Ampère's 'law'*. Again, the strange factor $\mu_0/4\pi$ is due to the historical way in which the electrical units were defined. The constant μ_0 is called the *permeability of the vacuum* and is defined by the fraction of newtons per ampere squared given in the formula. It is easy to see that the magnetic field has an intensity given by $\boldsymbol{vE}/c^2$, where $\boldsymbol{E}$ is the electric field measured by an observer moving with the charge. This is one of the many hints that magnetism is a relativistic effect.

We note that equation (?) is valid only for small velocities and accelerations. Can you find the general relation?

1.1 Electromagnetism

In 1831, Michael Faraday discovered an additional piece of the puzzle, one that even the great Ampère had overlooked. He found that a *moving* magnet could cause a current flow in an electrical circuit. Magnetism can thus be turned into electricity. This important discovery allowed the production of electrical current flow by generators, so-called *dynamos*, using water power, wind power or steam power. In fact, the first dynamo was built in 1832 by Ampère and his technician. Dynamos jump-started the use of electricity throughout the world. Behind every electrical wall plug there is a dynamo somewhere.

Oersted found that electric current can produce magnetic fields. Faraday found that magnetic fields could produce electric currents and electric fields. Electric and magnetic fields are two aspects of the same phenomenon: *electromagnetism*. It took another thirty years to unravel the full description.

Additional experiments show that magnetic fields also lead to electric fields when one changes to a moving viewpoint. You might check this on any of the examples of Figures 17 to 43. *Magnetism is relativistic electricity.* Electric and magnetic fields are partly transformed into each other when switching from one inertial reference frame to the other. Magnetic and electrical fields thus behave like space and time, which are also mixed up when changing from one inertial reference frame to the other. The theory of special relativity thus tells us that there must be a single concept, an *electromagnetic field*, describing them both. Investigating the details, one finds that the electromagnetic field $\boldsymbol{F}$ surrounding charged bodies has to be described by an antisymmetric 4-*tensor*

$$\boldsymbol{F}^{\mu v} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \text{or } \boldsymbol{F}_{\mu v} \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}. \tag{6}$$

Obviously, the electromagnetic field $\boldsymbol{F}$, and thus every component of these matrices, depends on space and time. The matrices show that electricity and magnetism are two faces of the same effect.[4] In addition, since electric fields appear only in the topmost row and leftmost column, the expressions show that in everyday life, for small speeds, electricity and magnetism *can* be separated. (Why?)

Using relativistic notation, the electromagnetic field is thus defined through the 4-acceleration $\boldsymbol{b}$ that it produces on a charge q of mass m and 4-velocity $\boldsymbol{u}$:

$$m\boldsymbol{b} = q\boldsymbol{Fu}$$

4. Actually, the expression for the field contains everywhere the expression $\sqrt{\mu_0 \varepsilon_0}$ instead of the speed of light c. We will explain the reason for this substitution shortly.

or, in 3-vector notation

$$\mathrm{d}E/\mathrm{d}t = q\boldsymbol{E}\boldsymbol{v} \,\text{and}\, \mathrm{d}\boldsymbol{p}/\mathrm{d}t = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{7}$$

2 Results on motion of quantum particles

Quantons, or quantum particles, differ from everyday particles: quantum particles *interfere*: they behave like a mixture of particles and waves. This property follows directly from the existence of $\hbar$, the smallest action in nature. From the existence of $\hbar$, quantum theory deduces all its statements about quantum particle motion. We summarize the main ones. There is no rest in nature. All objects obey the indeterminacy principle, which states that the indeterminacies in position x and momentum p follow

$$\Delta x \Delta p \geqslant \hbar/2 \,\text{with}\, \hbar = 1.1 \cdot 10^{34}\text{Js} \tag{8}$$

and making rest an impossibility. The state of quantum particles is defined by the same observables as in classical physics, with the difference that observables do not commute. Classical physics appears in the limit that the Planck constant $\hbar$ can effectively be set to zero.

Quantum theory introduces a *probabilistic element* into motion. It results from the minimum action value through the interactions with the baths that are part of the environment of every physical system.

Quantum particles *behave like waves*. The associated de Broglie wavelength λ is given by the momentum p through

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p} \tag{9}$$

both in the case of matter and of radiation. This relation is the origin of the wave behaviour of light and matter. The light particles are called photons; their observation is now standard practice. Quantum theory states that particle waves, like all waves, interfere, refract, disperse, dampen, can be dampened and can be polarized. This applies to photons, electrons, atoms and molecules. All waves being made of quantum particles, all waves can be seen, touched and moved. Light for example, can be 'seen' in photon-photon scattering, can be 'touched' using the Compton effect, and can be 'moved' by gravitational bending. Matter particles, such as molecules or atoms, can be seen in electron microscopes and can be touched and moved with atomic force microscopes. The interference and diffraction of wave particles is observed daily in the electron microscope.

Matter waves can be imagined as clouds that rotate locally. In the limit of negligible cloud size, quantum particles can be imagined as rotating little arrows.

Particles cannot be enclosed. Even though matter is impenetrable, quantum theory shows that tight boxes or insurmountable obstacles do not exist. Waiting long enough always allows us to overcome any boundary, since there is a finite probability to overcomeany obstacle. This process is called tunnelling when seen from the spatial point of view and is called decay when seen from the temporal point of view. Tunnelling explains the working of television tubes as well as radioactive decay.

All particles and all particle beams can be rotated. Particles possess an intrinsic angular momentum called spin, specifying their behaviour under rotations. Bosons have integer spin, fermions have half integer spin. An even number of bound fermions or any number of bound bosons yield a composite boson; an odd number of bound fermions or an infinite number of interacting bosons yield a low-energy fermion. Solids are impenetrable because of the fermion character of its electrons in the atoms.

Identical particles are indistinguishable. Radiation is made of indistinguishable particles called bosons, matter of fermions. Under exchange, fermions commute at space-like separations, whereas bosons anticommute. All other properties of quantum particles are the same as for classical particles, namely countability, interaction, mass, charge, angular momentum, energy, momentum, position, as well as impenetrability for matter and penetrability for radiation. Perfect copying machines do not exist.

In collisions, particles interact locally, through the exchange of other particles. When matter particles collide, they interact through the exchange of virtual bosons, i.e., offshell bosons. Motion change is thus due to particle exchange. Exchange bosons of even spinmediate only attractive interactions. Exchange bosons of odd spinmediate repulsive interactions as well.

The properties of collisions imply the existence of antiparticles, which are regularly observed in experiments. Elementary fermions, in contrast to many elementary bosons, differ from their antiparticles; they can be created and annihilated only in pairs. Elementary fermions have non-vanishing mass and move slower than light.

Images, made of radiation, are described by the same properties as matter. Images can only be localized with a precision of the wavelength λ of the radiation producing them. The appearance of Planck's constant $\hbar$ implies that length scales and time scales exist in nature. Quantum theory introduces a fundamental jitter in every example of motion. Thus the infinitely small is eliminated. In this way, lower limits to structural dimensions and to many other measurable quantities appear. In particular, quantum theory shows that it is impossible that on the electrons in an atom small creatures live in the same way that humans live on the Earth circling the Sun. Quantum theory shows the impossibility of Lilliput.

2.1 What is light?

Such a harmonic plane electromagnetic wave satisfies equation (?) for any value of *amplitude* A_0, of *phase* δ, and of *angular frequency* ω, provided the angular frequency and the *wave vector* $\boldsymbol{k}$ satisfy the relation

$$\omega(\boldsymbol{k}) = \frac{1}{\sqrt{\varepsilon_0\mu_0}}k \text{ or } \omega(\boldsymbol{k})\frac{1}{\sqrt{\varepsilon_0\mu_0}}\sqrt{\boldsymbol{k}^2}. \tag{10}$$

Theorem 1. *Let f be analytic in the region G except for the isolated singularities $a_1, a_2, ..., a_m$. If γ is a closed rectifiable curve in G which does not pass through any of the points a_k and if $\gamma \approx 0$ in G then*

$$\frac{1}{2\pi i}\int_\gamma f = \sum_{k=1}^{m} n(\gamma; a_k)\mathrm{Res}(f; a_k).$$

Theorem 2. *Let G be a bounded open set in $\mathbb{C}$ and suppose that f is a continuous function on G^- which is analytic in G. Then*

$$\max\{|f(z)|: z\varepsilon G^-\} = \max\{|f(z)|: z\varepsilon \partial G\}.$$

Thus, if we use pure quaternions such as V or W to describe positions, we can use unit quaternions to describe rotations and to calculate coordinate changes. The concatenation of two rotations is then given by the product of the corresponding unit quaternions. Indeed, a rotation by an angle α about the axis $\boldsymbol{l}$ followed by a rotation by an angle β about the axis $\boldsymbol{m}$ gives a rotation by an angle γ about the axis $\boldsymbol{n}$, with the values determined by

$$(\cos\gamma/2, \sin\gamma/2\boldsymbol{n}) = (\cos\beta/2, \sin\beta/2\boldsymbol{m})(\cos\alpha/2, \sin\alpha/2\boldsymbol{l}). \tag{11}$$

To calculate quantum motion with the principle of least action, we need to define the kinetic and the potential energy in terms of strands. There are various possibilities for Lagrangian densities for a given evolution equation; however, all are equivalent. In case of the free Schrödinger equation, one possibility is:

$$\mathcal{L} = \frac{\mathrm{i}\hbar}{2}(\bar{\psi}\partial_t\psi - \partial_t\bar{\psi}\psi) - \frac{\hbar^2}{2m}\nabla\bar{\psi}\,\nabla\psi \tag{12}$$

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Glossary

There's nothing.

Index

You want to generate that! You've manually generated it with Microsoft Excel.