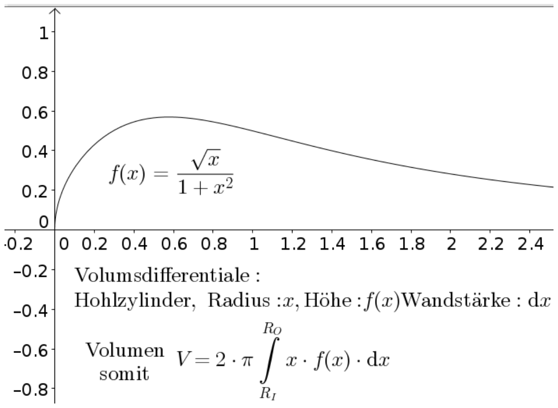

$I_V = \int x\, f(x)\, \mathrm{d}x$

$= \int \frac{x\,\sqrt{x}}{1+x^2}\, \mathrm{d}x$

$\boldsymbol{\sqrt{x} = u};\ \frac{1}{2\,\sqrt{x}}\,\mathrm{d}x = \mathrm{d}u$

$\mathrm{d}x = 2\,u\,\mathrm{d}u$

$I_V = \int \frac{u^3}{1+u^4}\cdot 2\,u\,\mathrm{d}u$

$I_V = 2\int \frac{u^4}{1+u^4}\,\mathrm{d}u$

$\frac{u^4}{1+u^4} = 1 - \frac{1}{1+u^4}$

$I_V = 2\,u - I$

zu bestimmen ist also noch

$I = \int \frac{1}{1+u^4}\,\mathrm{d}u$

$$
\begin{aligned}
I &= \int \frac{\mathrm{d}u}{(1+u^2)^2 - 2\,u^2} = \int \frac{\mathrm{d}u}{\left(1+\sqrt{2}\,u+u^2\right)\left(1-\sqrt{2}\,u+u^2\right)} \\
&= \int \left(\frac{a\,u+b}{1+\sqrt{2}\,u+u^2} + \frac{c\,u+d}{1-\sqrt{2}\,u+u^2}\right)\mathrm{d}u \\
&= \int \left(\frac{u^0\,(b+d) + u\left(a - b\,\sqrt{2} + c + d\,\sqrt{2}\right) + u^2\left(-a\,\sqrt{2} + b + c\,\sqrt{2} + d\right) + u^3\,(a+c)}{\left(1+\sqrt{2}\,u+u^2\right)\left(1-\sqrt{2}\,u+u^2\right)}\right)\mathrm{d}u
\end{aligned}
$$

Der u^0-Koeffitient muß =1 sein, alle übrigen =0; $\qquad w:=\sqrt{2}$

$$\left(\begin{array}{cccccc} & a & b & c & d & = \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z1 & 1 & -w & 1 & w & 0 \\ z2 & -w & 1 & w & 1 & 0 \\ z3 & 1 & 0 & 1 & 0 & 0 \end{array}\right) \quad z1 \leftrightarrows z0 \quad \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & -w & 1 & w & 1 & 0 \\ z3 & 1 & 0 & 1 & 0 & 0 \end{array}\right)$$

$$\begin{array}{c|ccccc} z2 & -w & 1 & w & 1 & 0 \\ +w\,z1 & +w & -2 & w & 2 & 0 \\ \hline = & 0 & -1 & 2\,w & 3 & 0 \\ \hline z3 & 1 & 0 & 1 & 0 & 0 \\ -z1 & -1 & w & -1 & -w & 0 \\ \hline = & 0 & w & 0 & -w & 0 \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & -1 & 2\,w & 3 & 0 \\ z3 & 0 & w & 0 & -w & 0 \end{array}\right)$$

$$\begin{array}{c|ccccc} z2 & 0 & -1 & 2w & 3 & 0 \\ +z0 & 0 & 1 & 0 & 1 & 1 \\ \hline = & 0 & 0 & 2\,w & 4 & 1 \\ \hline z3 & 0 & w & 0 & -w & 0 \\ +w\,z2 & 0 & -w & 4 & 3\,w & 0 \\ \hline = & 0 & 0 & 4 & 2\,w & 0 \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\,w & 4 & 1 \\ z3 & 0 & 0 & 4 & 2\,w & 0 \end{array}\right)$$

$$\begin{array}{c|ccccc} z3 & 0 & 0 & 4 & 2\,w & 0 \\ -w\,z2 & 0 & 0 & -4 & -4w & -w \\ \hline = & 0 & 0 & 0 & -2\,w & -w \end{array} \qquad \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\,w & 4 & 1 \\ z3 & 0 & 0 & 0 & -2\,w & -w \end{array}\right)$$

$$h:=\frac{1}{2}; \quad \frac{z3}{-2w} = \; 0 \;\; 0 \;\; 0 \;\; 1 \;\; h \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\,w & 4 & 1 \\ z3 & 0 & 0 & 0 & 1 & h \end{array}\right)$$

$$\begin{array}{c|ccccc} z2 & 0 & 0 & 2\,w & 4 & 1 \\ -4\,z3 & 0 & 0 & 0 & -4 & -2 \\ \hline = & 0 & 0 & 2\,w & 0 & -1 \\ \hline z1 & 1 & -w & 1 & w & 0 \\ -w\,z3 & 0 & 0 & 0 & -w & -w\,h \\ \hline = & 1 & -w & 1 & 0 & -w\,h \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & 0 & -w\,h \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 2\,w & 0 & -1 \\ z3 & 0 & 0 & 0 & 1 & h \end{array}\right) \quad z2:=\frac{z2}{2\,w} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & 0 & -w\,h \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 1 & 0 & -h/w \\ z3 & 0 & 0 & 0 & 1 & h \end{array}\right)$$

$$\begin{array}{c|ccccc} z1 & 1 & -w & 1 & 0 & -w\,h \\ +w\,z0 & & +w & & & +w\,h \\ -z2 & 0 & 0 & -1 & 0 & +h/w \\ \hline = & 1 & 0 & 0 & 0 & +h/w \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & 0 & 0 & 0 & +h/w \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 1 & 0 & -h/w \\ z3 & 0 & 0 & 0 & 1 & h \end{array}\right)$$

$$h/w=\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{4}\;; \quad \boxed{\begin{array}{cccccc} a & = & \dfrac{\sqrt{2}}{4} & b & = & \dfrac{1}{2} \\ c & = & \dfrac{-\sqrt{2}}{4} & d & = & \dfrac{1}{2} \end{array}}$$

$$\Rightarrow I \int \left(\frac{a\,u+b}{1+\sqrt{2}\,u+u^2}+\frac{c\,u+d}{1-\sqrt{2}\,u+u^2}\right)\mathrm{d}u \;=\; \frac{1}{4}\int\left(\frac{\sqrt{2}u+2}{1+\sqrt{2}\,u+u^2}+\frac{-\sqrt{2}\,u+2}{1-\sqrt{2}\,u+u^2}\right)\mathrm{d}u$$

$$=\frac{1}{4\sqrt{2}}\int\left(\frac{2\,u+2\sqrt{2}}{1+\sqrt{2}\,u+u^2}+\frac{-2\,u+2\sqrt{2}}{1-\sqrt{2}\,u+u^2}\right)\mathrm{d}u \;=\; \frac{1}{4\sqrt{2}}\int\left(\frac{2\,u+\sqrt{2}}{1+\sqrt{2}\,u+u^2}-\frac{2\,u-\sqrt{2}}{1-\sqrt{2}\,u+u^2}+K\right)\mathrm{d}u 2\,\pi$$

$$I \;=\; \frac{1}{4\sqrt{2}}\left(\ln\frac{1+\sqrt{2}\,u+u^2}{1-\sqrt{2}\,u+u^2}+\int K\,\mathrm{d}u\right)$$

$$\begin{aligned}
\text{mit}\quad \int K\,\mathrm{d}u &= \sqrt{2}\int\left(\frac{1}{1+\sqrt{2}\,u+u^2}+\frac{1}{1-\sqrt{2}\,u+u^2}\right)\mathrm{d}u\\
\int K\,\mathrm{d}u &= \sqrt{2}\int\left(\frac{1}{(u+\sqrt{2}/2)^2+1/2}+\frac{1}{(u-\sqrt{2}/2)^2+1/2}\right)\mathrm{d}u\\
\int K\,\mathrm{d}u &= 2\sqrt{2}\int\left(\frac{1}{2(u+\sqrt{2}/2)^2+1}+\frac{1}{2(u-\sqrt{2}/2)^2+1}\right)\mathrm{d}u\\
&= 2\sqrt{2}\int\left(\frac{1}{(\sqrt{2}\,u+1)^2+1}+\frac{1}{(\sqrt{2}\,u-1)^2+1}\right)\mathrm{d}u 2\,\pi\\
&= \frac{2\sqrt{2}}{\sqrt{2}}\left(\arctan(\sqrt{2}\,u+1)+\arctan(\sqrt{2}\,u-1)\right)\\
I &= \frac{1}{4\sqrt{2}}\left(\ln\frac{1+\sqrt{2}\,u+u^2}{1-\sqrt{2}\,u+u^2}+2\left(\arctan(\sqrt{2}\,u+1)+\arctan(\sqrt{2}\,u-1)\right)\right)\\
\text{Rücksubstitution } u &= \sqrt{x}\\
I &= \frac{1}{4\sqrt{2}}\left(\ln\frac{1+\sqrt{2\,x}+x}{1-\sqrt{2\,x}+x}+2\left(\arctan(\sqrt{2\,x}+1)+\arctan(\sqrt{2\,x}-1)\right)\right)\\
\text{somit}\quad V &= 2\,\pi\int_{R_I}^{R_O} x\cdot f(x)\cdot\mathrm{d}x\\
&= 2\,\pi\left|2\sqrt{x}-\frac{1}{4\sqrt{2}}\left(\ln\frac{1+\sqrt{2\,x}+x}{1-\sqrt{2\,x}+x}+2\left(\arctan(\sqrt{2\,x}+1)+\arctan(\sqrt{2\,x}-1)\right)\right)\right|_{R_I}^{R_O}
\end{aligned}$$

Verfaßt

mit texmacs (http://www.texmacs.org),

und Bild am Beginn mit geogebra (http://www.geogebra.org).

Bild endet am rechten Rand der Kurve und unter dem R_I der Formel $V=2\cdot\pi\int_{R_I}^{R_O} x\cdot f(x)\cdot\mathrm{d}x$,

Formel im Bild sind geogebra-gerendertes handcodiertes Latex