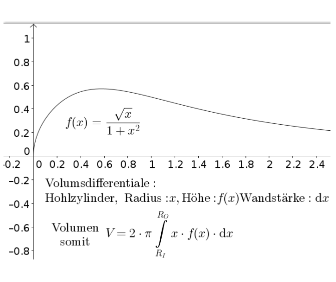

$$I_V = \int x\, f(x)\, \mathrm{d}x$$
$$= \int \frac{x\sqrt{x}}{1+x^2}\, \mathrm{d}x$$
$$\boldsymbol{\sqrt{x} = u};\ \tfrac{1}{2\sqrt{x}}\, \mathrm{d}x = \mathrm{d}u$$
$$\mathrm{d}x = 2\, u\, \mathrm{d}u$$
$$I_V = \int \tfrac{u^3}{1+u^4} \cdot 2\, u\, \mathrm{d}u$$
$$I_V = 2 \int \frac{u^4}{1+u^4}\, \mathrm{d}u$$
$$\tfrac{u^4}{1+u^4} = 1 - \tfrac{1}{1+u^4}$$
$$I_V = 2\, u - I$$

zu bestimmen ist also noch

$$I = \int \frac{1}{1+u^4}\, \mathrm{d}u$$

$$\begin{aligned}
I &= \int \frac{\mathrm{d}u}{(1+u^2)^2 - 2\, u^2} = \int \frac{\mathrm{d}u}{\left(1+\sqrt{2}\, u+u^2\right)\left(1-\sqrt{2}\, u+u^2\right)} \\
&= \int \left(\frac{a\, u+b}{1+\sqrt{2}\, u+u^2} + \frac{c\, u+d}{1-\sqrt{2}\, u+u^2} \right) \mathrm{d}u \\
&= \int \left(\frac{u^0\,(b+d) + u\left(a - b\sqrt{2} + c + d\sqrt{2}\right) + u^2\left(-a\sqrt{2} + b + c\sqrt{2} + d\right) + u^3\,(a+c)}{\left(1+\sqrt{2}\, u+u^2\right)\left(1-\sqrt{2}\, u+u^2\right)} \right) \mathrm{d}u
\end{aligned}$$

Der u^0 – Koeffitient muß =1 sein, alle übrigen =0; $\quad w := \sqrt{2}$

$$\left(\begin{array}{cccccc} & a & b & c & d & = \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z1 & 1 & -w & 1 & w & 0 \\ z2 & -w & 1 & w & 1 & 0 \\ z3 & 1 & 0 & 1 & 0 & 0 \end{array} \right) \ z1 \leftrightarrows z0 \ \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & -w & 1 & w & 1 & 0 \\ z3 & 1 & 0 & 1 & 0 & 0 \end{array} \right)$$

$$\begin{array}{c|ccccc} z2 & -w & 1 & w & 1 & 0 \\ +w\, z1 & +w & -2 & w & 2 & 0 \\ \hline = & 0 & -1 & 2\, w & 3 & 0 \\ \hline z3 & 1 & 0 & 1 & 0 & 0 \\ -z1 & -1 & w & -1 & -w & 0 \\ \hline = & 0 & w & 0 & -w & 0 \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & -1 & 2\, w & 3 & 0 \\ z3 & 0 & w & 0 & -w & 0 \end{array} \right)$$

$$\begin{array}{c|ccccc} z2 & 0 & -1 & 2w & 3 & 0 \\ +z0 & 0 & 1 & 0 & 1 & 1 \\ \hline = & 0 & 0 & 2\, w & 4 & 1 \\ \hline z3 & 0 & w & 0 & -w & 0 \\ +w\, z2 & 0 & -w & 4 & 3\, w & 0 \\ \hline = & 0 & 0 & 4 & 2\, w & 0 \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\, w & 4 & 1 \\ z3 & 0 & 0 & 4 & 2\, w & 0 \end{array} \right)$$

$$\begin{array}{c|ccccc} z3 & 0 & 0 & 4 & 2\, w & 0 \\ -w\, z2 & 0 & 0 & -4 & -4w & -w \\ \hline = & 0 & 0 & 0 & -2\, w & -w \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\, w & 4 & 1 \\ z3 & 0 & 0 & 0 & -2\, w & -w \end{array} \right)$$

$$h := \frac{1}{2};\quad \frac{z3}{-2w} = \ 0\ \ 0\ \ 0\ \ 1\ \ h \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & w & 0 \\ z0 & 0 & 1 & 0 & 1 & 1 \\ z2 & 0 & 0 & 2\, w & 4 & 1 \\ z3 & 0 & 0 & 0 & 1 & h \end{array} \right)$$

$$\begin{array}{c|ccccc} z2 & 0 & 0 & 2\, w & 4 & 1 \\ -4\, z3 & 0 & 0 & 0 & -4 & -2 \\ \hline = & 0 & 0 & 2\, w & 0 & -1 \\ \hline z1 & 1 & -w & 1 & w & 0 \\ -w\, z3 & 0 & 0 & 0 & -w & -w\, h \\ \hline = & 1 & -w & 1 & 0 & -w\, h \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & 0 & -w\, h \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 2\, w & 0 & -1 \\ z3 & 0 & 0 & 0 & 1 & h \end{array} \right) \ z2 := \frac{z2}{2\, w} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & -w & 1 & 0 & -w\, h \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 1 & 0 & -h/w \\ z3 & 0 & 0 & 0 & 1 & h \end{array} \right)$$

$$\begin{array}{c|ccccc} z1 & 1 & -w & 1 & 0 & -w\, h \\ +w\, z0 & & +w & & & +w\, h \\ -z2 & 0 & 0 & -1 & 0 & +h/w \\ \hline = & 1 & 0 & 0 & 0 & +h/w \end{array} \Rightarrow \left(\begin{array}{cccccc} & a & b & c & d & = \\ z1 & 1 & 0 & 0 & 0 & +h/w \\ z0 & 0 & 1 & 0 & 0 & h \\ z2 & 0 & 0 & 1 & 0 & -h/w \\ z3 & 0 & 0 & 0 & 1 & h \end{array} \right)$$

$$h/w = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4};\ \boxed{\begin{array}{llll} a = & \frac{\sqrt{2}}{4} & b = & \frac{1}{2} \\ c = & \frac{-\sqrt{2}}{4} & d = & \frac{1}{2} \end{array}}$$

$$\begin{aligned}
\Rightarrow I \int \left(\frac{a\, u+b}{1+\sqrt{2}\, u+u^2} + \frac{c\, u+d}{1-\sqrt{2}\, u+u^2} \right) \mathrm{d}u &= \frac{1}{4} \int \left(\frac{\sqrt{2} u+2}{1+\sqrt{2}\, u+u^2} + \frac{-\sqrt{2}\, u+2}{1-\sqrt{2}\, u+u^2} \right) \mathrm{d}u \\
= \frac{1}{4\sqrt{2}} \int \left(\frac{2\, u+2\sqrt{2}}{1+\sqrt{2}\, u+u^2} + \frac{-2\, u+2\sqrt{2}}{1-\sqrt{2}\, u+u^2} \right) \mathrm{d}u &= \frac{1}{4\sqrt{2}} \int \left(\frac{2\, u+\sqrt{2}}{1+\sqrt{2}\, u+u^2} - \frac{2\, u-\sqrt{2}}{1-\sqrt{2}\, u+u^2} + K \right) \mathrm{d}u 2\pi \\
I &= \frac{1}{4\sqrt{2}} \left(\ln \frac{1+\sqrt{2}\, u+u^2}{1-\sqrt{2}\, u+u^2} + \int K\, \mathrm{d}u \right) \\
\text{mit} \quad \int K\, \mathrm{d}u &= \sqrt{2} \int \left(\frac{1}{1+\sqrt{2}\, u+u^2} + \frac{1}{1-\sqrt{2}\, u+u^2} \right) \mathrm{d}u \\
\int K\, \mathrm{d}u &= \sqrt{2} \int \left(\frac{1}{(u+\sqrt{2}/2)^2+1/2} + \frac{1}{(u-\sqrt{2}/2)^2+1/2} \right) \mathrm{d}u \\
\int K\, \mathrm{d}u &= 2\sqrt{2} \int \left(\frac{1}{2(u+\sqrt{2}/2)^2+1} + \frac{1}{2(u-\sqrt{2}/2)^2+1} \right) \mathrm{d}u \\
&= 2\sqrt{2} \int \left(\frac{1}{(\sqrt{2}\, u+1)^2+1} + \frac{1}{(\sqrt{2}\, u-1)^2+1} \right) \mathrm{d}u 2\pi \\
&= \frac{2\sqrt{2}}{\sqrt{2}} \left(\arctan(\sqrt{2}\, u+1) + \arctan(\sqrt{2}\, u-1) \right) \\
I &= \frac{1}{4\sqrt{2}} \left(\ln \frac{1+\sqrt{2}\, u+u^2}{1-\sqrt{2}\, u+u^2} + 2\left(\arctan(\sqrt{2}\, u+1) + \arctan(\sqrt{2}\, u-1) \right) \right) \\
\text{Rücksubstitution } u &= \sqrt{x} \\
I &= \frac{1}{4\sqrt{2}} \left(\ln \frac{1+\sqrt{2\, x}+x}{1-\sqrt{2\, x}+x} + 2\left(\arctan(\sqrt{2\, x}+1) + \arctan(\sqrt{2\, x}-1) \right) \right) \\
\text{somit} \quad V &= 2\pi \int_{R_I}^{R_O} x \cdot f(x) \cdot \mathrm{d}x \\
&= 2\pi \left| 2\sqrt{x} - \frac{1}{4\sqrt{2}} \left(\ln \frac{1+\sqrt{2\, x}+x}{1-\sqrt{2\, x}+x} + 2\left(\arctan(\sqrt{2\, x}+1) + \arctan(\sqrt{2\, x}-1) \right) \right) \right|_{R_I}^{R_O}
\end{aligned}$$

Verfaßt

mit texmacs (http://www.texmacs.org),

und Bild am Beginn mit geogebra (http://www.geogebra.org).

Bild endet am rechten Rand der Kurve und unter dem R_I der Formel $V = 2 \cdot \pi \int_{R_I}^{R_O} x \cdot f(x) \cdot \mathrm{d}x$,

Formel im Bild sind geogebra-gerendertes handcodiertes Latex